The Phillips Curve in the United States and Canada: A GARCH-DCC Analysis

Lu Yang[a] and Shigeyuki Hamori[b,*]

[a]*Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada-Ku, Kobe 657-8501, Japan*

[b]*Faculty of Economics, Kobe University, 2-1 Rokkodai, Nada-Ku, Kobe 657-8501, Japan*

Abstract: By applying the GARCH-DCC model, we reexamine the Phillips curve based on a time-varying correlation analysis for Canada and the United States from January 1985 to December 2012. The empirical results show that the sign of the correlation between the inflation rate and the unemployment rate is negative during recession periods but positive during boom periods.

Keywords: GARCH-DCC model, Phillips curve, financial crisis.

1. INTRODUCTION

The Phillips curve, which characterizes the negative relationship between the inflation rate and the unemployment rate, is considered to be one of the most important stylized facts in macroeconomics. Empirical evidence of UK wage behavior was originally provided by Phillips (1958) and interpreted theoretically by Lipsey (1960), offering policymakers the choice between inflation and unemployment. Since then, numerous empirical studies have been provided and the estimated Phillips curve has been found to be rather unstable over time and its use not justified for a long time span (see Friedman, 1968; Mankiw, 2000; Rubio *et al*., 2007).

In particular, numerous studies have examined the inflation–unemployment relation for the US economy. For example, Karanassou and Sala (2010) state that the US Phillips curve is not vertical even in the longrun. Their results imply that the nominal and real sides of the economy are symbiotic. Moreover, Sachsida *et al*. (2011) find important long-run co-movement between inflation and unemployment in the US economy. Further, Cevik and Dibooglu (2013) find that although shocks to US unemployment dissipate in expansions, shocks to the unemployment rate seem to be persistent in recessions.

In contrast to the studies above, we investigate the dynamic correlation between inflation and unemployment by considering the autoregressive conditional heteroskedasticity (ARCH) effect. A notable addition to the body of knowledge on this topic was the recent study of Phillips curves by Russell and Chowdhury (2013), which retrieved the standard empirical results of Phillips curves based on a GARCH (generalized autoregressive conditional heteroskedasticity) model with structural breaks. Rather than focus on the stable relationship between the inflation rate and the unemployment rate in the long-term, however, the present paper allows for time-varying correlations in this relation by considering time series properties such as serial correlations, the ARCH process, and fat tails. The presence of these properties can prevent traditional econometric methods from accurately describing the Phillips curve. Therefore, we employ the GARCH-DCC (dynamic conditional correlation) model (Rahman and Serletis, 2012; Jones and Olson, 2013) to solve the discussed problem and select Canada and the United States to be our case studies for comparison and contrasting purposes. In contrast to the study of Russell and Chowdhury (2013), the present paper focuses on an empirical methodology by using DCC to examine the Phillips curves in these countries.

2. METHODOLOGY

We follow the study of Hamilton (2008) by modeling the series presented herein as a GARCH process. Further, we apply Engle's (2002) DCC techniques to describe the time-varying correlations between the inflation rate and the unemployment rate.

Let $y_t = [y_1, y_{2t}]'$ be a 2×1 vector that contains the data series on the inflation rate and the unemployment rate. The VAR (vector autoregression) can be expressed as follows:

$$A(L)y_t = \varepsilon_t \tag{1}$$

*Address correspondence to this author at the Faculty of Economics, Kobe University, 2-1 Rokkodai, Nada-Ku, Kobe 657-8501, Japan;
Tel: +81 0788036832; E-mail: hamori@econ.kobe-u.ac.jp

where *A* (*L*) is a matrix in the lag operator *L* and $\varepsilon_t = \left[\varepsilon_{1t}, \varepsilon_{2t}\right]'$ is a vector of innovations. Equation 1, the VAR (*k*) model, indicates that the current movement of variables y_t can be explained by their own past movements $(y_{t-1}, y_{t-2}, \cdots)$.

The innovation process is ruled by the following GARCH (*p*, *q*) process:

$$h_{i,t} = \omega_i + \sum_{i=1}^{p} \alpha_i \varepsilon_{i,t-1}^2 + \sum_{i=1}^{q} \beta_i h_{i,t-1} (i = 1, 2) \tag{2}$$

where E_{t-1} is the conditional information operator based on the information at time *t*-1.

We apply DCC to illustrate the dynamic relationship between the inflation rate and the unemployment rate. Specifically, based on the conditional volatilities from Equation 2, we calculate the conditional correlations from the conditional covariance matrix as

$$H_t = E[\varepsilon_t \varepsilon_t'] = D_t R_t D_t \tag{3}$$

where the diagonal matrix D_t represents the conditional volatilities from Equation 2.

Engle (2002) considers a dynamic matrix process

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B) + A'z_{t-1}z_{t-1}'A + B'Q_{t-1}B \tag{4}$$

where $\bar{Q}$ represents the unconditional correlation matrices of z_t.Then, the conditional correlation matrix R_t is derived as

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{5}$$

where the diagonal matrix $Q_t^* = \sqrt{q_{ii,t}}$ contains the square roots of the diagonal elements of Q_t.

In particular, the bivariate DCC can be specified as

$$q_{ij,t} = (\bar{q} - a^2\bar{q} - b^2\bar{q}) + a^2 z_{ij,t-1} + b^2 q_{ij,t-1} \tag{6}$$

where $z_{i,t-1}$ is the time-varying vector of standardized residuals $\frac{\varepsilon_{i,t-1}}{h_{i,t-1}}$ and the restriction condition is $a^2 + b^2 < 1$.

3. DATA

To analyze the Phillips curve of price and unemployment, we employ the monthly consumer price index (CPI) and unemployment rate in Canada and the United States from January 1985 to December 2012. The inflation rate is computed as 1200 times the log monthly change in the CPI. All seasonally adjusted data series are derived from DataStream. The descriptive statistics are reported in Table **1**.

Table 1: Descriptive Statistics for the Inflation Rate and the Unemployment Rate

	The United States		Canada	
	Inflation	**Unemployment**	**Inflation**	**Unemployment**
Mean	2.801	6.099	2.262	8.304
Median	2.793	5.700	2.184	7.900
Maximum	16.409	10.000	9.477	12.100
Minimum	−21.437	3.800	−3.898	5.900
Std. Dev.	3.138	1.510	2.059	1.514
Skewness	−1.563	0.849	0.258	0.520
Kurtosis	15.663	3.010	3.867	2.330
Jarque–Bera	2381.690	40.396	14.240	21.426
Unit root test (ADF)	−11.681***	−2.297	−3.274**	−1.516
Zivot–Andrews Test		−4.665**		−6.489***
Q(12)	77.173***	3479.784***	145.400***	3490.459***
Obs	336	336	336	336

Notes: ** and *** represent significance at the 5% and 1% levels, respectively. *Q*(12) is the Ljung–Box *Q* statistics for the null hypothesis that there is no autocorrelation up to order 12 for standardized residuals.
Source: DataStream.

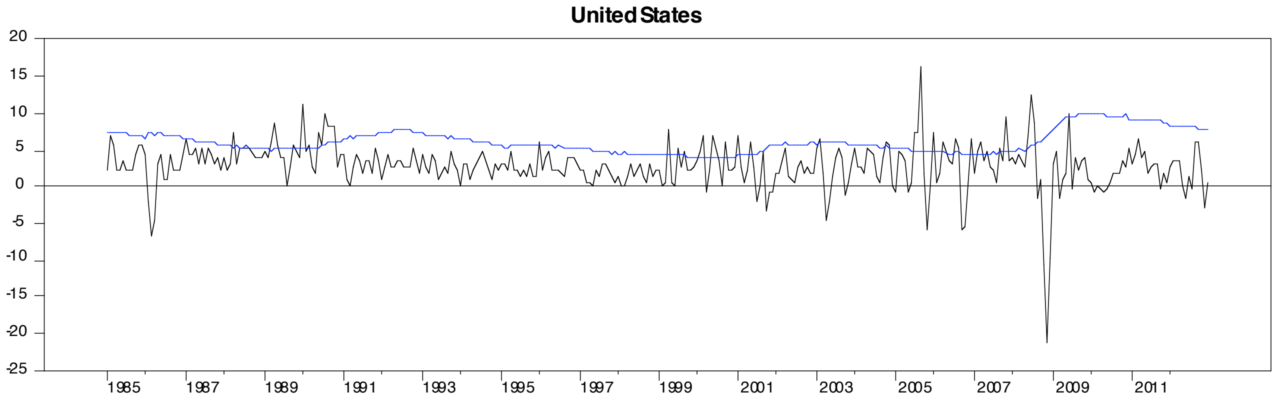

Figure 1: Time series plots of the inflation rate (black) and the unemployment rate (blue).

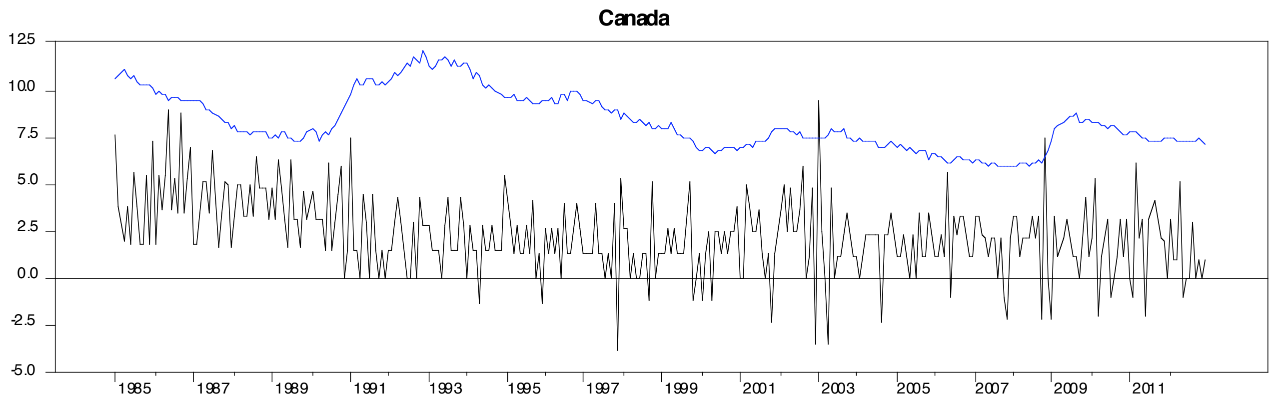

Figure 2: Time series plots of the inflation rate (black) and the unemployment rate (blue).

Based on augmented Dickey–Fuller (ADF) tests, the inflation rate is found to be stationary, while the unemployment rate contains a unit root. However, since Perron's (1989) analyses show that structural breaks can lead to erroneously accepting unit roots, we also implement the Zivot and Andrews (1992) unit root test for the unemployment rate. This test indicates that the unemployment rate is stationary in levels with a structural break occurring in November 2008 for Canada and in May 2008 for the United States. To account for these structural breaks, we incorporate a dummy variable (D = 1) into Equation 1 for all $t \geq$ November 2008 in Canada and for all $t \geq$ May 2008 in the United States. The results of the Jarque–Bera test indicate that the null hypothesis of normality is rejected for all cases, which indirectly supports the existence of ARCH effects. The results of the Ljung–Box Q statistics demonstrate that the null hypothesis of no autocorrelation up to order 12 is rejected at the 1% level for all cases. These data series are plotted in Figures **1** and **2**.

4. EMPIRICAL RESULTS

The results are shown in Table **2**. Most of the estimates in the variance equation are statistically significant at the 10% level, and they satisfy the restrictions of $\omega > 0, \alpha > 0, \beta > 0, \alpha + \beta < 1,$ which confirms that GARCH-type models are appropriate. The coefficients in the DCC model are also estimated to be statistically significant at the 10% level, and these also satisfy the restrictions of $a^2+b^2<1$. The Ljung–Box Q statistics also suggest that the empirical results of the models have been adequately estimated.

Figures **3** and **4** display the time-varying correlations from the estimated models. The correlation between the inflation rate and the unemployment rate tends to range from −0.25 to 0.15 for the United States and from −0.24 to 0.13 for Canada. During the economic recession and contraction periods (i.e., the 2000s for Canada and 2001–2004 for the United States), a negative correlation between the inflation rate and the unemployment rate consistently exists. However, a continuous positive relationship between

Table 2: Bivariate GARCH-DCC Model

	The United States		Canada	
	π_t	u_t	π_t	u_t
Mean equation				
C	2.419 (0.588)***	0.178 (0.051)***	0.164 (0.619)	0.054 (0.060)
π_{t-1}	0.512 (0.057)***	0.001 (0.003)	−0.118 (0.057)**	−0.003 (0.005)
π_{t-2}	−0.199 (0.064)***	−0.003 (0.003)	0.070 (0.057)	−0.006 (0.005)
π_{t-3}	0.015 (0.065)	−0.003 (0.003)	0.093 (0.057)*	0.002 (0.005)
π_{t-4}	0.059 (0.065)	0.003 (0.003)	0.204 (0.056)***	−0.002 (0.005)
π_{t-5}	−0.118 (0.065)*	−0.002 (0.003)	0.116 (0.057)**	0.001 (0.005)
π_{t-6}	0.058 (0.066)	0.002 (0.003)	0.142 (0.056)**	0.013 (0.004)***
π_{t-7}	0.014 (0.064)	0.004 (0.003)	0.123 (0.057)**	−0.004 (0.005)
π_{t-8}	−0.059 (0.066)	0.002 (0.003)	−0.007 (0.058)	0.001 (0.005)
π_{t-9}	0.041 (0.066)	0.004 (0.003)	0.080 (0.056)	−0.000 (0.005)
π_{t-10}	0.006 (0.066)	0.003 (0.003)		
π_{t-11}	0.157 (0.065)***	−0.001 (0.003)		
π_{t-12}	−0.214 (0.058)***	0.003 (0.003)		
u_{t-1}	−0.327 (1.134)	0.899 (0.048)***	−0.466 (0.586)	0.957 (0.069)***
u_{t-2}	−1.432 (1.544)	0.134 (0.050)***	1.617 (0.840)**	0.001 (0.087)
u_{t-3}	0.825 (1.555)	0.112 (0.050)**	−0.783 (0.844)	0.108 (0.078)***
u_{t-4}	0.553 (1.567)	−0.121 (0.056)**	−0.466 (0.831)	−0.016 (0.070)
u_{t-5}	−0.546 (1.569)	0.082 (0.045)*	0.398 (0.827)	0.143 (0.069)**
u_{t-6}	0.787 (1.567)	−0.058 (0.036)	−0.734 (0.827)	−0.166 (0.066)***
u_{t-7}	0.628 (1.566)	0.050 (0.050)	−0.626 (0.827)	−0.017 (0.067)
u_{t-8}	0.182 (1.554)	−0.071 (0.060)	1.245 (0.827)	−0.020 (0.069)
u_{t-9}	0.114 (1.548)	−0.026 (0.043)	−0.129 (0.578)	0.002 (0.047)
u_{t-10}	−1.057 (1.541)	−0.036 (0.046)		
u_{t-11}	−0.752 (1.534)	0.128 (0.036)***		
u_{t-12}	0.885 (1.113)	−0.124 (0.038)***		
D		0.134 (0.032)***		−0.009 (0.026)
Variance equation				
ω	0.232 (0.144)*	0.013 (0.001)***	1.029 (0.480)**	0.012 (0.002)***
α	0.367 (0.076)***	0.005 (0.045)	0.129 (0.069)***	0.341 (0.087)***
β	0.673 (0.058)***	0.198 (0.075)***	0.567 (0.182)***	0.301 (0.079)***

(Table 2). Continued.

	The United States		Canada	
	π_t	u_t	π_t	u_t
DCC				
a^2	0.052 (0.030)*		0.0330(0.013)***	
b^2	0.946 (0.031)***		0.9668(0.013)***	
Diagnostic				
Q(12)	7.509 [0.584]	1.297 [0.998]	6.185 [0.721]	10.534 [0.309]
Q(16)	11.080 [0.604]	4.769 [0.980]	8.675 [0.797]	15.035 [0.305]
Q(20)	18.250 [0.373]	16.829 [0.465]	17.612 [0.414]	17.455 [0.424]
Q^2(12)	14.736 [0.098]	12.860 [0.169]	8.309 [0.503]	11.552 [0.239]
Q^2(16)	15.442 [0.281]	20.382 [0.086]	16.614 [0.217]	12.587 [0.480]
Q^2(20)	17.392 [0.428]	21.078 [0.222]	19.395 [0.306]	15.283 [0.575]

Notes: π_t and u_t denote the inflation rate and the unemployment rate at time *t*, respectively. The numbers in parentheses are standard errors. The numbers in square brackets are *p*-values. *Q*(12), *Q*(16), and *Q*(20) (Q^2(12), Q^2(16), and Q^2(20)) are the Ljung–Box *Q* statistics for the null hypothesis that there is no autocorrelation up to orders 12, 16, and 20 for standardized residuals (standardized squared residuals), respectively. *, **, and *** represent significance at the 10%, 5%, and 1% levels, respectively.

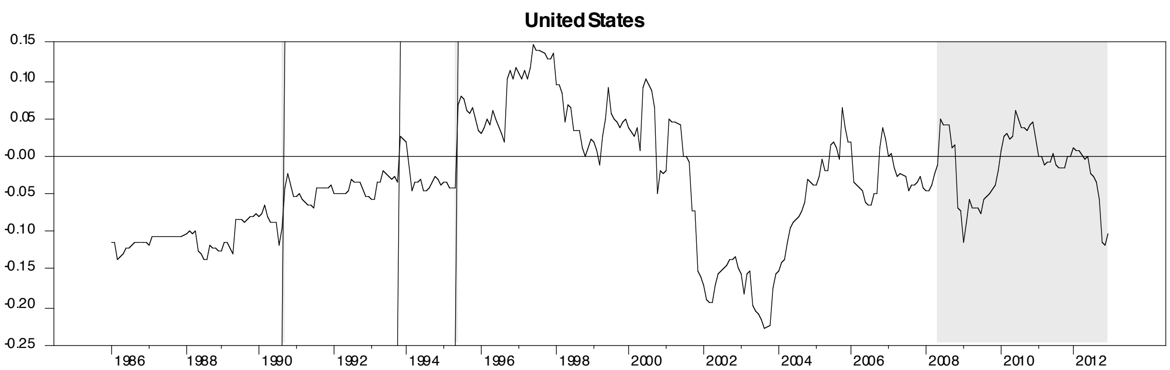

Figure 3: Conditional correlation between the inflation rate and the unemployment rate.

Notes: The shaded portion of the figure represents the dates after the structural break in the unemployment rate. The dashed lines from left to right denote the 1990 oil price spike, Black Wednesday in 1992, and the economic crisis in Mexico in 1994, respectively.

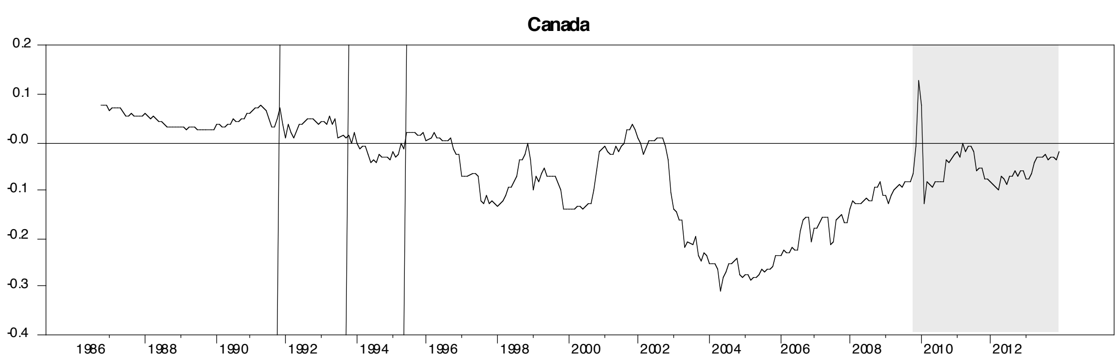

Figure 4: Conditional correlation between the inflation rate and the unemployment rate.

Notes: The shaded portion of the figure represents the dates after the structural break in the unemployment rate. The dashed lines from left to right denote the 1990 oil price spike, Black Wednesday in 1992, and the economic crisis in Mexico in 1994, respectively.

the inflation rate and the unemployment rate is observed in the United States from 1995 to 1999 due to the IT bubble, and the same phenomenon is detected in Canada from 1985 to 1989 due to the Toronto bubble.

Consistent with the findings of Sachsida *et al*. (2011) and Cevik and Dibooglu (2013), the Phillips curves estimated in this study proved to be rather unstable in the long run for the economies of both Canada and the United States. In contrast to the findings of previous studies, however, we show that a negative inflation–unemployment relationship does not exist during economic booms. Moreover, monetary policy, perhaps by stimulating inflation, seems to be more effective at reducing unemployment during recession periods.

5. CONCLUSION

By applying the GARCH-DCC model, we reexamined the Phillips curves of price and unemployment for the United States and Canada from January 1985 to December 2012. Compared with previous studies, the empirical results found a negative correlation during periods of contraction and recession but a positive correlation during periods of relatively rapid economic growth. For example, the IT bubble in the United States from 1995 to 1999 and Toronto bubble in Canada from 1985 to 1989 exhibited positive correlations between the inflation rate and the unemployment rate.

Our results have at least two implications for policymakers. First, monetary policy may have more important and long-lasting effects on unemployment during a recession period compared with during a tranquil period. Second, since keeping the unemployment rate at a reasonable level is the priority of monetary authorities, the tradeoff between inflation and unemployment must be evaluated before implementing any policy, especially when the economy is booming.

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